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SEQUENTIAL EQUILIBRIUM DETECTION AND REPORTING
POLICIES IN A MODEL OF TAX EVASION

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ABSTRACT

Noncompliance with tax laws and other forms of criminal activity have typically been treated as equivalent; both have been modeled in decision-theoretic terms, with the same probability of detection applying to all agents. However, noncompliance with tax laws is different from other criminal activities because taxpayers are required to submit a preliminary accounting of their behavior, while potential criminals obviously are not. This preliminary round of information transmission differentiates individuals, and raises the possibility that it may not be optimal to apply the same probability of detection to all taxpayers.

We develop a game-theoretic model of income tax compliance in which the taxpayer possesses private information about his own income, while the IRS knows only the probability distribution according to which the taxpayer's income is realized. By investing effort, the IRS can (stochastically) verify a taxpayer's income. We characterize the sequential equilibrium for this game, which consists of a reporting rule for the individual taxpayer, and a verification policy for the IRS.

Our equilibrium has the feature that taxpayers with greater true income under-report less than those with lower true income, and efforts at verification are lower the greater is reported income. If individuals can be classified on the basis of some observable characteristic which is related to opportunities for income, we find that classes of taxpayers who enjoy greater opportunities for high income under-report to a greater extent; accordingly, more effort is devoted to their investigation. This is to be distinguished from the former result, which applies to different types of taxpayers within the same class.

SEQUENTIAL EQUILIBRIUM VERIFICATION AND REPORTING POLICIES

IN A MODEL OF TAX COMPLIANCE*

Jennifer F. Reinganum and Louis L. Wilde

I. Introduction

There is a substantial literature devoted to tax compliance and, more generally, to the economics of crime and punishment. These problems have typically been treated as equivalent; they have both been modeled as portfolio problems, in which an agent must allocate his budget (of income, or effort) between the risky asset (unreported income, or criminal activity) and the risk-free asset (reported income, or legal activity) (see e.g., Allingham and Sandmo, 1972; Becker, 1968; Polinsky and Shavell, 1979; Srinivasan, 1973; and Stigler, 1970). Each of these analyses assumes a fixed probability that proscribed behavior will be detected, although some attention has been given to the determination of the optimal probability of detection. More recently, there have been repeated-games analyses of these problems (Landsberger and Meilijson, 1982; Greenberg, 1984; and Rubinstein, 1979), but again these models are equally applicable to tax compliance and other criminal activity.

We consider the problem of tax compliance to be fundamentally different from that of other criminal activities because taxpayers are required to submit a preliminary accounting of their behavior, while potential criminals obviously face no such requirement. This preliminary round of information transmission will tend to differentiate individuals, and raises the possibility that it may not be optimal to apply the same verification policy to all taxpayers. In this paper, we incorporate the information content of the income reporting process into an equilibrium model of tax

compliance and enforcement.

To this end, we assume the taxpayer possesses private information about his own income level, while the IRS knows only the probability distribution according to which the taxpayer's income is realized. By investing resources, the IRS can (perhaps only stochastically) verify the taxpayer's income. We assume the cost of verification is dependent upon the probability of verification chosen by the IRS. We seek to characterize the optimal reporting rule for an individual taxpayer, given his private information, and given the verification policy of the IRS. Similarly, we wish to characterize the optimal verification policy of the IRS, given the reporting behavior of taxpayers, and given its incomplete information regarding the taxpayer's true income.

Our approach to this problem represents an application of the sequential equilibrium solution concept (Kreps and Wilson, 1982a). This methodology has yielded interesting results when applied to problems in limit pricing (Milgrom and Roberts, 1982a; Matthews and Mirman, 1983; and Saloner, 1983), reputation and predation (Kreps and Wilson, 1982b; Milgrom and Roberts, 1982b), bargaining (Fudenberg and Tirole, 1983; and Cramton, 1984), signalling (Spence, 1974; Kreps, 1984; Milgrom and Roberts, 1984) and convertible debt call policy (Harris and Raviv, 1984). Other recent analyses of related problems can be found in Baron and Besanko (1983), Reinganum and Wilde (forthcoming) and Townsend (1979). These three papers do not use the sequential equilibrium approach adopted here; instead they employ a principal-agent structure to examine the problem of optimal auditing.

In Section II we set up our basic game-theoretic model of tax compliance, and examine necessary conditions for equilibrium. Two candidates for equilibrium are identified and characterized. In Section III we verify

that one candidate actually does constitute a sequential equilibrium for the game described in Section II, and we rule out the other candidate. This equilibrium has the feature that taxpayers with greater true income under-report less than those with lower true income, and efforts at verification are lower the greater is reported income. In Section IV we present an algebraic example. In Section V we use a comparative static analysis to examine the impact of separating taxpayers into classes on the basis of some characteristic which is directly observable. We find that classes of taxpayers who enjoy greater opportunities for high income under-report to a greater extent; accordingly, more effort is devoted to their detection. This is to be distinguished from the former result, which applies to different types of taxpayers within the same class. In Section VI we consider the impact of declining verification costs upon the equilibrium verification and reporting policies. Section VII reconsiders the analysis of Section II under the assumption that taxpayers are risk averse. Section VIII concludes and suggests possibilities for future research.

II. The Model

The timing of moves in the model are as follows: the taxpayer observes his true income; we will often refer to the taxpayer's true income as the taxpayer's "type." Based upon true income, the taxpayer conveys a statement of reported income to the IRS. Since the IRS does not observe the taxpayer's true income, it must make some conjecture about the type of taxpayer who would report a given level of income. Based on the level of income reported and these conjectures, the IRS chooses a level of effort to be devoted to investigating the taxpayer. This effort will be assumed to generate a particular probability that the taxpayer's true income will be verified, with the property that greater effort leads to a greater probability

of verification. Thus investigation does not imply certain apprehension.

Suppose that true (taxable) income for the taxpayer is a random variable $I \in [\underline{I}, \bar{I}]$, where $\underline{I} < \bar{I} < \infty$, with distribution function $F(\cdot)$. Let x denote reported income for a taxpayer. A strategy for the taxpayer is a reporting policy $x = r(I)$, where $r: [\underline{I}, \bar{I}] \rightarrow (-\infty, \infty)$; that is, the taxpayer may report any level of income, not just those which might possibly occur. If the taxpayer is not investigated, then he is asked to pay a tax of t_x dollars; that is, we assume proportional taxation. If the taxpayer is investigated and his true income is ascertained, then a tax plus a fine proportional to evaded tax is assessed: if I is the taxpayer's true income, this amount is $tI + t\pi(I-x)$. Note that no feasibility restrictions are placed on the tax assessments; individuals who report negative income and are not investigated receive a transfer (or negative income tax), and taxpayers whose payment exceeds their income due to over-reporting or a fine are bound by their reports, and hence suffer negative income.

Given the taxpayer's report, the IRS must choose a level of effort to devote to investigation. Since this effort yields a probability of verification which is monotonically increasing in effort, we can treat the IRS as choosing a probability of verification, with an associated effort cost. Let p denote the probability of verification. A strategy for the IRS is a verification policy $p = p(x)$, where $p: (-\infty, \infty) \rightarrow [0, 1]$. Since any report is permissible, the verification policy must be defined for any possible report. Let $c(p)$ denote the cost to the IRS of sustaining probability p of verification. We assume that $c(0) = 0$, and that $c(\cdot)$ is twice continuously differentiable for $p \in (0, 1)$ and satisfies the following restrictions: for $p \in (0, 1)$,

$$(A1) \quad 0 < c'(p) < \infty \text{ and } 0 < c''(p) < \infty.$$

$$(A2) \quad \lim_{\rho \rightarrow 1} c'(\rho) = \infty; \text{ and}$$

$$(A3) \quad c'(\rho)/c''(\rho) + \rho > 1/(1 + \pi).$$

Restriction (A3) is a curvature condition which ensures that the marginal cost of verification does not rise too quickly. The point at which this restriction is subsequently used will be emphasized.

Finally, since the IRS does not observe I directly, it must form beliefs or conjectures which relate reports to types of taxpayers. Let $\mu(d|x)$ denote the IRS' prior probability assessment that the true type of a taxpayer who reports x belongs to the set $d \subseteq [I, \bar{I}]$. We require that $\mu([I, \bar{I}]|x) = 1$; that is, the IRS's beliefs cannot assign to any report a taxpayer type which is known to be nonexistent.

Throughout this section we assume that both the IRS and the taxpayer are risk-neutral, and maximize expected net revenue and expected net income, respectively. Risk aversion on the part of taxpayers is considered in Section VII. Expected net revenue to the IRS when it observes a report of x and chooses a probability ρ of verification, conditional upon its beliefs $\mu(d|x)$, is

$$R(x, \rho; \mu) = \rho[tE_{\mu}(I|x) + \pi t(E_{\mu}(I|x) - x)] + (1 - \rho)tx - c(\rho),$$

where $E_{\mu}(I|x)$ represents the expected value of the taxpayer's income, given that he reported income x , based upon the IRS' prior probability assessment μ ; that is, $E_{\mu}(I|x) = \int_{[I, \bar{I}]} Id\mu(I|x)$.

Expected net income to a taxpayer who has true income I and reports income x , conditional upon the verification policy $p(\cdot)$, is

$$N(I, x; p) = p(x)[I - tI - t\pi(I - x)] + (1 - p(x))(I - tx).$$

Note that the taxpayer bears no costs when being investigated, suffering a penalty only if noncompliance is ascertained. The functions $R(x, \rho; \mu)$ and $N(I, x; p)$ represent the payoffs to the IRS and the taxpayer, respectively.

Definition 1. A triple $(\bar{\mu}(d|\cdot), \bar{p}(\cdot), \bar{r}(\cdot))$ is a sequential equilibrium if

(a) Given the beliefs $\bar{\mu}(d|\cdot)$, $\bar{p}(x)$ maximizes $R(x, \rho; \bar{\mu})$ (sequential rationality);

(b) Given the verification policy $\bar{p}(\cdot)$, $\bar{r}(I)$ maximizes $N(I, x; \bar{p})$; and

(c) $\bar{\mu}(d|x) = \mu_F(d \cap \bar{r}^{-1}(x)) / \mu_F(\bar{r}^{-1}(x))$ for all $d \subseteq [I, \bar{I}]$,

where $\mu_F(d) = \int_d dF(I)$ (consistency).

This definition admits the possibility of pooling equilibria, in which $\bar{r}^{-1}(x)$ is set-valued. However, we will be specifically interested in a separating equilibrium, in which $\bar{r}(I)$ is monotonically increasing; in this case, $\bar{r}^{-1}(x)$ is single-valued. Consequently, we define separating beliefs, which assign a unique taxpayer type to each report x . Let $\tau: (-\infty, \infty) \rightarrow [I, \bar{I}]$ denote these beliefs. Given the beliefs $\tau(\cdot)$, we can rewrite the expected net revenue to the IRS as follows.

$$R(x, \rho; \tau) = \rho[t\tau(x) + \pi t(\tau(x) - x)] + (1 - \rho)tx - c(\rho).$$

Definition 2. A triple $(\bar{\tau}(\cdot), \bar{p}(\cdot), \bar{r}(\cdot))$ is a separating sequential equilibrium if

(a) Given the beliefs $\bar{\tau}(x)$, $\bar{p}(x)$ maximizes $R(x, \rho; \bar{\tau})$ (sequential rationality);

(b) Given the verification policy $\bar{p}(x)$, $\bar{r}(I)$ maximizes $N(I, x; \bar{p})$; and

(c) $\bar{\tau}(\bar{r}(I)) = I$ for all $I \in [I, \bar{I}]$ (consistency).

Alternatively, a consistency condition equivalent to (c) is

$$\bar{\tau}(x) = \bar{r}^{-1}(x) \text{ for all } x \in [\bar{r}(\underline{I}), \bar{r}(\bar{I})].$$

Necessary Conditions for a Separating Equilibrium

The IRS maximizes $R(x, p; \tau)$ by a choice of $p = p(x)$. Pointwise optimization requires that

$$R_p(x, p(x); \tau) = t(1 + \pi)(\tau(x) - x) - c'(p(x)) \geq 0 \ (\leq 0) \text{ and } p(x) \leq 1 \ (\geq 0). \quad (1)$$

When $p(x)$ is interior, the necessary condition is

$$R_p(x, p(x); \tau) = t(1 + \pi)(\tau(x) - x) - c'(p(x)) = 0. \quad (1')$$

Since $c''(p) > 0$ for all p , equation (1) is necessary and sufficient to determine the optimal verification policy $p(x)$ (given $\tau(x)$).

The taxpayer maximizes $N(I, x; p)$ by a choice of $x = r(I)$. If $p(\cdot)$ is differentiable, then the optimal report (given $p(x)$) solves

$$N_x(I, r(I); p) = p'(r(I))[-t(1 + \pi)(I - r(I))] + p(r(I))t\pi - t(1 - p(r(I))) = 0. \quad (2)$$

If $p(\cdot)$ is twice differentiable, then a second-order necessary condition is

$$N_{xx}(I, r(I); p) = p''(r(I))[-t(1 + \pi)(I - r(I))] + 2p'(r(I))t(1 + \pi) \leq 0. \quad (3)$$

Equations (1), (2) and (3) hold simultaneously at a sequential equilibrium. Incorporating the consistency condition that $I = \tau(x) = r^{-1}(x)$, we can re-write equations (1'), (2) and (3) as follows:

$$t(1 + \pi)(r^{-1}(x) - x) - c'(p(x)) = 0, \quad (4)$$

$$p'(x)[-t(1 + \pi)(r^{-1}(x) - x)] + p(x)t\pi - t(1 - p(x)) = 0, \quad (5)$$

$$p''(x)[-t(1 + \pi)(r^{-1}(x) - x)] + 2p'(x)t(1 + \pi) \leq 0. \quad (6)$$

Equations (4) and (5) can be combined to give equation (7) below, an ordinary differential equation:

$$-p'(x)c'(p(x)) + t\pi p(x) - t(1 - p(x)) = 0. \quad (7)$$

The ordinary differential equation (7) has two kinds of solutions. First, there is a one-parameter family of solutions, $p_0(\cdot)$, in which $p_0(x)$ depends non-trivially on x . We can rewrite (7) as follows:

$$p' = [t\pi p - t(1 - p)]/c'(p)$$

The expression $[t\pi p - t(1 - p)]/c'(p)$ is continuously differentiable in p on $[0, 1)$ under assumption (A1). It then follows that for any given boundary condition $p_0(a) = b$, where $a \in (-\infty, \infty)$ and $b \in [0, 1)$, a unique solution $p_0(x)$ to (7) exists (at least in a neighborhood of $x = a$). Moreover, $p_0(x)$ will be twice differentiable (Hestenes, p. 49, Theorem 14.1). Second, because equation (7) is nonlinear, there is also a singular solution $\tilde{p}(x) = 1/(1 + \pi)$, in which the verification policy is independent of reported income. The

solutions $p_0(\cdot)$ and $\tilde{p}(\cdot)$ imply (inverse) reporting policies $r_0^{-1}(\cdot)$ and $\tilde{r}^{-1}(\cdot)$, respectively, which can be obtained by solving equation (4). Finally, we need to add appropriate beliefs. Consistency requires that $\tau_0(x) = r_0^{-1}(x)$ for $x \in [r_0(\underline{I}), r_0(\bar{I})]$; the "natural" beliefs associated with $p_0(x)$ for reports outside the range of those which would be reported in equilibrium by any existing type are: $\tau_0(x) = \bar{I}$ for $x > r_0(\bar{I})$, and $\tau_0(x) = \underline{I}$ for $x < r_0(\underline{I})$. (The

natural beliefs associated with $\tilde{p}(x)$ are analogously derived.) These beliefs assign the nearest type to reports which lie outside the range of equilibrium reports which would be made by any existing type of taxpayer. We will refer to a solution of equation (7), along with its associated reporting policy and beliefs, as a candidate for equilibrium if it also satisfies equation (6); that is, if the second-order necessary condition for the taxpayer's optimum is satisfied.

Note that equation (7) is merely suggestive of candidates for a sequential equilibrium; these candidates must be verified, modified or eliminated by checking whether the implied verification and reporting policies actually are best against each other.

Consider the triple (τ_0, p_0, r_0) . Because $p_0(x)$ is twice differentiable, $r_0^{-1}(x)$ will also be differentiable. By construction, the pair (p_0, r_0^{-1}) satisfy equations (4) and (5) for all x . Equations (4) and (5) can be differentiated to obtain:

$$t(1 + \pi)(r_0^{-1}'(x) - 1) - c''(p_0)p_0'(x) = 0 \quad (8)$$

and

$$p_0''(x)[-t(1 + \pi)(r_0^{-1}(x) - x)] + 2p_0'(x)t(1 + \pi) - p_0'(x)t(1 + \pi)r_0^{-1}'(x) = 0. \quad (9)$$

Lemma 1. If $(p_0(x), r_0^{-1}(x))$ satisfies (6) with a strict inequality, then $p_0'(x) < 0$ and $r_0^{-1}'(x) \in (0, 1)$.

Proof. From equation (8), either (i) $p_0'(x) > 0$ and $r_0^{-1}'(x) - 1 > 0$, or (ii) $p_0'(x) < 0$ and $r_0^{-1}'(x) - 1 < 0$. From equation (9) and assuming that inequality (6) is strict, either (iii) $p_0'(x) > 0$ and $r_0^{-1}'(x) < 0$ or (iv)

$p_0'(x) < 0$ and $r_0^{-1}'(x) > 0$. Since (i) and (iii) are mutually inconsistent, it follows that (ii) and (iv) hold. That is, $p_0'(x) < 0$ and $r_0^{-1}'(x) \in (0, 1)$.

Q.E.D.

Lemma 1 implies that one candidate for an equilibrium verification policy has the property that a taxpayer who reports greater income faces a lower probability of verification. Since $r_0^{-1}'(x) = 1/r_0'(I)$, it follows that $r_0'(I) > 1$; that is, under-reported income $I - r_0(I)$ declines with true income. To see why this combination makes intuitive sense, note that $p_0'(x) < 0$ gives taxpayers an incentive to report higher income, while $r_0'(I) > 1$ gives the IRS a greater incentive to investigate those who report low income. Another (possibly surprising) feature of this equilibrium is that verification and reporting policies do not depend on the precise form of the income distribution function $F(\cdot)$, they only depend on its support $[I, \bar{I}]$.

III. A Constructive Approach to Equilibrium

In this Section, we use a constructive approach to characterize equilibrium for cost functions which satisfy assumptions (A1), (A2) and (A3).

Define $\bar{x} = \bar{I} - c'(0)/t(1 + \pi)$. Next solve the ordinary differential equation (7) using $p(\bar{x}) = 0$ as a boundary condition. Denote this solution by $p_0(x)$. Next define $r_0^{-1}(x) = x + c'(p_0(x))/t(1 + \pi)$. Note that r_0^{-1} is differentiable. If $p_0(x)$ satisfies the following condition (B), then the pair (p_0, r_0^{-1}) satisfies equations (4), (5) and (6) (with a strict inequality).

$$(B) \quad -p_0''(x)c'(p_0(x)) + 2p_0'(x)t(1 + \pi) < 0 \text{ for all } x \leq \bar{x}.$$

Lemma 1 then implies that $p_0'(x) < 0$ and $r_0^{-1}'(x) \in (0, 1)$. Thus $r_0^{-1}(x)$ is invertible to obtain $x = r_0(I)$. Define $\underline{x} \in (-\infty, \bar{x}]$ such that $\underline{x} = r_0(\underline{I})$.

Since $r_0^{-1}(\cdot)$ is monotone increasing, if \underline{x} exists, then it will be unique and $\underline{x} < \bar{x}$; if no such value exists, define $\underline{x} \equiv -\infty$.

Theorem 1. If $p_0(x)$ exists throughout $[\underline{x}, \bar{x}]$ and satisfies condition (B), then the following triple is a sequential equilibrium. The equilibrium verification and reporting policies are illustrated in Figure 1.

(i) The equilibrium verification policy is

$$\bar{p}(x) = \begin{cases} 0 & x \geq \bar{x} \\ p_0(x) & x \in [\underline{x}, \bar{x}] \\ c'^{-1}(t(1 + \pi)(\underline{I} - x)) & x \leq \underline{x} \end{cases}$$

(ii) The equilibrium reporting policy $\bar{r}(I)$ is the unique value of $x \in [\underline{x}, \bar{x}]$ such that

$$I = \bar{r}(I) + c'(p_0(\bar{r}(I)))/t(1 + \pi);$$

that is, $\bar{r}(I) = r_0(I)$ as defined above.

(iii) Finally, the equilibrium beliefs are

$$\bar{\tau}(x) = \begin{cases} \bar{I} & x \geq \bar{x} \\ r_0^{-1}(x) & x \in [\underline{x}, \bar{x}] \\ \underline{I} & x \leq \underline{x} \end{cases}$$

Proof. Since $\bar{r}(\bar{I}) = r_0(\bar{I}) = \bar{x}$ and $\bar{r}(\underline{I}) = r_0(\underline{I}) = \underline{x}$ and since r_0 is invertible, $\bar{\tau}(x)$ satisfies the consistency requirement.

Given $\bar{\tau}(\cdot)$, we next show that $\bar{p}(x)$ maximizes $R(x, p; \bar{\tau})$. For $x \geq \bar{x}$, $\bar{\tau}(x) = \bar{I}$. Then

$$R_p(x, 0; \bar{\tau}) = t(1 + \pi)(\bar{I} - x) - c'(0) < 0 \text{ for all } x > \bar{x},$$

since $R_p(\bar{x}, 0; \bar{\tau}) = 0$. Thus $\bar{p}(x) = 0$ for $x \geq \bar{x}$. For $x \in [\underline{x}, \bar{x}]$, $\bar{\tau}(x) = r_0^{-1}(x)$. Then

$$R_p(x, 0; \bar{\tau}) = t(1 + \pi)(r_0^{-1}(x) - x) - c'(0) = c'(p_0(x)) - c'(0).$$

Since $p_0(\bar{x}) = 0$ and $p_0'(x) < 0$, $p_0(x) > 0$ for all $x < \bar{x}$. Thus $c''(\cdot) > 0$ implies that $c'(p_0(x)) - c'(0) > 0$ for all $x < \bar{x}$. Thus $\bar{p}(x)$ is interior for $x < \bar{x}$. Solving $R_p(x, \bar{p}(x); \bar{\tau}) = 0$ yields $c'(p_0(x)) - c'(\bar{p}(x)) = 0$, or

$$\bar{p}(x) = p_0(x) \text{ for } x \in [\underline{x}, \bar{x}].$$

For $x \leq \underline{x}$, $\bar{\tau}(x) = \underline{I}$. Then

$$R_p(\underline{x}, 0; \bar{\tau}) = t(1 + \pi)(\underline{I} - \underline{x}) - c'(0) > 0$$

and $R_p(x, 0; \bar{\tau})$ is decreasing in x , so $R_p(x, 0; \bar{\tau}) > 0$ for all $x \leq \underline{x}$. Solving

$$R_p(x, \bar{p}(x); \bar{\tau}) = t(1 + \pi)(\underline{I} - x) - c'(\bar{p}(x)) = 0$$

yields $\bar{p}(x) = c'^{-1}(t(1 + \pi)(\underline{I} - x))$ for $x \leq \underline{x}$.

We now show that, given $\bar{p}(\cdot)$, $\bar{r}(I)$ maximizes $N(I, x; \bar{p})$. Note that N is continuous in x because $\bar{p}(\cdot)$ is continuous. Clearly any report of $x > \bar{x}$ is dominated by a report of $x = \bar{x}$. This is because the taxpayer is not investigated at all, and thus pays a tax based only on reported income. For $x < \underline{x}$, N is differentiable with

$$\begin{aligned} N_x(I, x; \bar{p}) &= \bar{p}'(x)[-t(1 + \pi)(\underline{I} - x)] + t\pi\bar{p}(x) - t(1 - \bar{p}(x)) \\ &= c'^{-1}'(t(1 + \pi)(\underline{I} - x))[t(1 + \pi)]^2(\underline{I} - x) + t\pi\bar{p}(x) - t(1 - \bar{p}(x)). \end{aligned}$$

Since $t(1 + \pi)(\underline{I} - x) = c'(\bar{p}(x))$ and since $c'^{-1}(c'(\bar{p}(x))) = 1/c''(\bar{p}(x))$, we can evaluate N_x at \underline{I} to obtain

$$N_x(\underline{I}, x; \bar{p}) = t(1 + \pi)c'(\bar{p}(x))/c''(\bar{p}(x)) + t(1 + \pi)\bar{p}(x) - t > 0$$

by (A3). Since $N_x(I, x; \bar{p})$ is increasing in I , $N_x(I, x; \bar{p}) > 0$ for $x < \underline{x}$, for all $I \in [\underline{I}, \bar{I}]$. Thus any report $x < \underline{x}$ is dominated by a report of $x = \underline{x}$. Finally, for reports $x \in [\underline{x}, \bar{x}]$, N is differentiable with

$$N_x(I, x; \bar{p}) = p'_0(x)[-t(1 + \pi)(I - x)] + t\pi p_0(x) - t(1 - p_0(x)).$$

Note that $N_x(\underline{I}, \underline{x}; \bar{p}) = 0$ because $\underline{I} - \underline{x} = c'(p_0(\underline{x}))/t(1 + \pi)$ and because $p_0(x)$ satisfies equation (7). Since $N_x(I, x; \bar{p})$ is increasing in I , $N_x(I, \underline{x}; \bar{p}) > 0$ for all $I > \underline{I}$. Similarly, $N_x(\bar{I}, \bar{x}; \bar{p}) = 0$, and since N_x is increasing in I , $N_x(I, \bar{x}; \bar{p}) < 0$ for all $I < \bar{I}$. Thus $\bar{r}(I) \in (\underline{x}, \bar{x})$ for all $I \in (\underline{I}, \bar{I})$. Solving $N_x(I, \bar{r}(I); \bar{p}) = 0$ for $\bar{r}(I)$ yields

$$\bar{r}(I) = I - [p_0(\bar{r}(I))t\pi - t(1 - p_0(\bar{r}(I)))/p'_0(\bar{r}(I))t(1 + \pi)].$$

Because $p_0(\cdot)$ satisfies equation (7), this reduces to

$$\bar{r}(I) = I - c'(p_0(\bar{r}(I)))/t(1 + \pi).$$

Q.E.D.

Let $T(I)$ denote the expected tax paid by a taxpayer with income I . Since only reports $x \in [\bar{r}(\underline{I}), \bar{r}(\bar{I})]$ will be observed in equilibrium, it follows that $\bar{p}(x) = p_0(x)$ for all observed reports x . Then the expected tax for a type I taxpayer can be written as follows.

$$T(I) = p_0(r_0(I))[tI + t\pi(I - r_0(I))] + (1 - p_0(r_0(I)))tr_0(I).$$

Corollary 1. For the sequential equilibrium verification and reporting policies of Theorem 1, $dT(I)/dI > 0$ and $d(T(I)/I)/dI < 0$ for all $I \in [\underline{I}, \bar{I}]$. That is, the expected tax $T(I)$ increases and the expected average tax rate $T(I)/I$ decreases with an increase in true income I . Thus the effective tax schedule in the presence of incomplete information $T(I)$ is regressive, although the statutory tax schedule is linear.

Proof. Recall that, by definition, $p_0(x)$ satisfies equation (7) for all x .

$$\begin{aligned} dT(I)/dI &= r'_0[p'_0 t(1 + \pi)(I - r_0) - t\pi p_0 + t(1 - p_0)] + t(1 + \pi)p_0 \\ &= r'_0[p'_0 c'(p_0) - t\pi p_0 + t(1 - p_0)] + t(1 + \pi)p_0 \\ &= t(1 + \pi)p_0, \end{aligned}$$

where p_0 and p'_0 are evaluated at $r_0(I)$, and r_0 and r'_0 are evaluated at I . The second equality follows from equation (4), and the third from equation (7). Thus $dT(I)/dI = t(1 + \pi)p_0 > 0$ for all $I \in [\underline{I}, \bar{I}]$.

$$\begin{aligned} d(T(I)/I)/dI &= [It(1 + \pi)p_0 - T(I)]/I^2 \\ &= [t\pi p_0 - t(1 - p_0)]r_0/I^2 \\ &= c'(p_0)p'_0 r_0/I^2 < 0 \end{aligned}$$

for all $I \in [\underline{I}, \bar{I}]$, where the second equality follows by substitution of $T(I)$ and the third follows from equation (7).

Q.E.D.

To see that the singular solution $\tilde{p} = 1/(1 + \pi)$ and its implied

reporting policy $\tilde{r}(I)$ do not constitute a sequential equilibrium for cost functions which satisfy our domain restrictions, recall that we have ruled out cost functions with the property that $c'(0) = 0$. This was done initially to ensure that equation (7) had a unique solution for the given boundary condition; $c'(0) = 0$ is also ruled out by (A3).

Solving equation (4) for the (inverse) reporting policy associated with the singular solution $\tilde{p}(x) = 1/(1 + \pi)$ yields

$$\tilde{r}^{-1}(x) - x = c'(1/(1 + \pi))/t(1 + \pi).$$

Let $\alpha = c'(1/(1 + \pi))/t(1 + \pi)$. Then the putative equilibrium has $\tilde{r}(I) = I - \alpha$ for $I \in [\underline{I}, \bar{I}]$, and $\tilde{r}(x) = x + \alpha$ for $x \in [\underline{I} - \alpha, \bar{I} - \alpha]$, while $\tilde{r}(x) = \underline{I}$ for $x \leq \underline{I} - \alpha$ and $\tilde{r}(x) = \bar{I}$ for $x \geq \bar{I} - \alpha$. To calculate the remainder of the putative equilibrium, we compute the optimal verification policy $\bar{p}(x)$ given $\tilde{r}(x)$.

Again let $\bar{x} = \bar{I} - c'(0)/t(1 + \pi)$. Then for $x > \bar{I} - \alpha$, $\tilde{r}(x) = \bar{I}$. Since

$$R_p(x, 0; \tilde{r}) = t(1 + \pi)(\bar{I} - x) - c'(0) < 0$$

for all $x > \bar{x}$, it follows that $\bar{p}(x) = 0$ for $x \geq \bar{x}$, while $\bar{p}(x)$ solves

$$R_p(x, \bar{p}(x); \tilde{r}) = t(1 + \pi)(\bar{I} - x) - c'(\bar{p}(x)) = 0$$

for $x \in [\bar{I} - \alpha, \bar{x}]$. This set is non-empty because $c''(\cdot) > 0$ implies that $\bar{x} = \bar{I} - c'(0)/t(1 + \pi) > \bar{I} - c'(1/(1 + \pi))/t(1 + \pi) = \bar{I} - \alpha$. Thus

$\bar{p}(x) = c'^{-1}(t(1 + \pi)(\bar{I} - x))$ for $x \in [\bar{I} - \alpha, \bar{x}]$. For $x \in [\underline{I} - \alpha, \bar{I} - \alpha]$, $\bar{p}(x) = \tilde{p}(x) = 1/(1 + \pi)$. Finally, for $x \leq \underline{I} - \alpha$, $\tilde{r}(x) = \underline{I}$, so $\bar{p}(x)$ solves

$$R_p(x, \bar{p}(x); \tilde{r}) = t(1 + \pi)(\underline{I} - x) - c'(\bar{p}(x)) = 0$$

or $\bar{p}(x) = c'^{-1}(t(1 + \pi)(\underline{I} - x))$ for $x \leq \underline{I} - \alpha$.

To see that this combination does not constitute a sequential equilibrium for cost functions satisfying our domain restrictions, note that $N(I, \tilde{r}(I); \bar{p}) = I(1 - t)$, while $N(I, \tilde{r}; \bar{p}) = I - t\bar{x}$. Thus $N(I, \tilde{r}; \bar{p}) > N(I, \tilde{r}(I); \bar{p})$ for all $I \in (\bar{I} - c'(0)/t(1 + \pi), \bar{I}]$. Since we require $c'(0) > 0$, this set is non-empty. Thus a non-empty subset of taxpayer types would prefer to deviate from the "equilibrium" reporting policy $\tilde{r}(I)$ and to convey a report which falls outside the interval of "equilibrium" reports $[\tilde{r}(\underline{I}), \tilde{r}(\bar{I})] = [\underline{I} - \alpha, \bar{I} - \alpha]$.

IV. An Example

In this section, we compute the equilibrium verification and reporting functions $\bar{p}(x)$ and $\tilde{r}(I)$ for a specific cost function. Assume that $c(p) = -c \ln(1 - p)$. This cost function satisfies (A1), (A2) and (A3).

The upper limit $\bar{x} = \bar{I} - c/t(1 + \pi)$. To complete the solution we need only solve equation (7) using as a boundary condition that $p_0(\bar{x}) = 0$ and verify that p_0 satisfies condition (B).

Solving equation (7) with this boundary condition yields

$$p_0(x) = 1 - \pi/[1 + \pi - \exp\{-(t\pi/c)(\bar{x} - x)\}].$$

The associated (inverse) reporting policy is

$$r_0^{-1}(x) = x + \frac{1 + \pi - \exp\{-(t\pi/c)(\bar{x} - x)\}}{(t\pi/c)(1 + \pi)}.$$

The lower limit \underline{x} is defined implicitly by $\underline{I} = r_0^{-1}(\underline{x})$. To see that \underline{x} exists and is unique, note that $r_0^{-1}(\bar{x}) = \bar{I}$, $\lim_{x \rightarrow -\infty} r_0^{-1}(x) = -\infty$, and $r_0^{-1}'(x) > 0$.

for all $x \leq \bar{x}$. Therefore there exists a unique value $\underline{x} \in (-\infty, \bar{x})$ such that $\underline{I} = r_0^{-1}(\underline{x})$.

It is tedious but straightforward to check that

$$-p_0''(x)c'(p_0(x)) + 2p_0'(x)t(1 + \pi) = \frac{-(t\pi)^2 \exp\{-(t\pi/c)(\bar{x} - x)\}}{c[1 + \pi - \exp\{-(t\pi/c)(\bar{x} - x)\}]} < 0$$

for all $x \leq \bar{x}$. That is, condition (B) holds for all $x \leq \bar{x}$.

Corollary 2. Let $c(\rho) = -c \ln(1 - \rho)$. Then the following triple is a sequential equilibrium.

(i) The verification policy is given by

$$\bar{p}(x) = \begin{cases} 0 & x \geq \bar{x} \\ 1 - \pi/[1 + \pi - \exp\{-(t\pi/c)(\bar{x} - x)\}] & x \in [\underline{x}, \bar{x}] \\ 1 - c/t(1 + \pi)(\underline{I} - x) & x \leq \underline{x} \end{cases}$$

(ii) The reporting policy $\bar{r}(I)$ is the unique value of $x \in [\underline{x}, \bar{x}]$ such that

$$I = \bar{r}(I) + \frac{1 + \pi - \exp\{-(t\pi/c)(\bar{x} - \bar{r}(I))\}}{(t\pi/c)(1 + \pi)}, \quad I \in [\underline{I}, \bar{I}];$$

that is, $\bar{r}(I) = r_0(I)$.

(iii) The beliefs which assign a taxpayer type I to each observed report x are given by

$$\bar{\tau}(x) = \begin{cases} \bar{I} & x \geq \bar{x} \\ r_0^{-1}(x) & x \in [\underline{x}, \bar{x}] \\ \underline{I} & x \leq \underline{x} \end{cases}$$

Proof. Corollary 2 follows directly from Theorem 1; an alternative direct proof is contained in the Appendix.

V. Different Classes of Taxpayers

We have seen in Sections III and IV that when equilibrium verification and reporting policies are (at least partially) characterized by equation (7), then the extent of under-reporting, $I - \bar{r}(I)$, falls as true income I rises, and the verification policy $\bar{p}(x)$ declines as reported income x rises.

However, our analysis assumed that all individuals (or types of taxpayers) were ex ante indistinguishable. If some other characteristic of individuals is observable, then the IRS can, in some cases, condition its verification policy upon this observable characteristic. Such characteristics may be relatively immutable (e.g., sex or race), or subject to choice but largely determined by other considerations (e.g., occupation or place of residence). For instance, suppose that two individuals, one residing in Beverly Hills and the other in Death Valley, report the same taxable income x . Since the ex ante distribution of income opportunities is likely to differ in a predictable way between these two locales, the equilibrium verification policy (and consequently the equilibrium reporting policy) should differentiate between these two identical reports.

To model this, we can consider changing \bar{I} , \underline{I} or both. For simplicity, suppose that \underline{I} is unchanged. We will refer to taxpayers with different values of \bar{I} as belonging to different classes of taxpayers; types of taxpayers will then vary within each class.

The following Lemma, which characterizes the relative positions of the functions $p(x) = p_0(x)$ and $p(x) = c^{-1}(t(1 + \pi)(I - x))$, will prove useful later.

Lemma 2. Suppose $p_0(x)$ is a solution to equation (7) which also satisfies condition (B). Then

- (a) $p_0(x) > c'^{-1}(t(1 + \pi)(\bar{I} - x))$ if $x > \underline{x}$;
- (b) $p_0(x) = c'^{-1}(t(1 + \pi)(\bar{I} - x))$ if $x = \underline{x}$; and
- (c) $p_0(x) < c'^{-1}(t(1 + \pi)(\bar{I} - x))$ if $x < \underline{x}$.

Proof. $p(x) = p_0(x)$ solves equation (1') with $\tau(x) = r_0^{-1}(x)$ and $p(x) = c'^{-1}(t(1 + \pi)(\bar{I} - x))$ solves (1') with $\tau(x) = \bar{I}$. The solution to (1') is increasing with inferred income $\tau(x)$. Since $r_0^{-1}(x)$ is monotone increasing with $r_0^{-1}(\underline{x}) = \bar{I}$, it follows that $r_0^{-1}(x) > \bar{I}$ if $x > \underline{x}$, $r_0^{-1}(x) = \bar{I}$ if $x = \underline{x}$, and $r_0^{-1}(x) < \bar{I}$ if $x < \underline{x}$. The claim follows.

Q.E.D.

We can now consider the effect of increasing \bar{I} upon the equilibrium verification and reporting policies. To do this, explicitly denote this dependence as follows: $\bar{p}(x; \bar{I})$ and $\bar{r}(I; \bar{I})$. A solution to equation (7) through 0 at $\bar{x} = \bar{I} - c'(0)/t(1 + \pi)$ is denoted $p_0(x; \bar{I})$. Since the solution to (7) is unique through any given value of $p \in [0, 1]$, $p_0(x; \bar{I})$ and $p_0(x; \bar{I}')$ cannot cross, for any $\bar{I} \neq \bar{I}'$. Since $p_0'(x; \bar{I}) < 0$, it follows that $p_0(x; \bar{I})$ is increasing in \bar{I} . That is, if $\bar{I}' > \bar{I}$, then $p_0(x; \bar{I}') > p_0(x; \bar{I})$ for all $x < \bar{x}' = \bar{I}' - c'(0)/t(1 + \pi)$.

Recall that the inverse reporting policy

$$r_0^{-1}(x; \bar{I}) = x + c'(p_0(x; \bar{I}))/t(1 + \pi)$$

is increasing; that is, $r_0^{-1}'(x; \bar{I}) > 0$. The reporting policy $\bar{r}(I; \bar{I})$ is defined by $r_0^{-1}(\bar{r}(I; \bar{I}); \bar{I}) - I = 0$. Since $r_0^{-1}'(x; \bar{I}) > 0$ and $r_0^{-1}(x; \bar{I})$ is increasing in \bar{I} , it follows that $\bar{r}(I; \bar{I})$ is decreasing in \bar{I} . Thus for any

given level of true income I , reported income falls as \bar{I} rises; alternatively put, unreported income $I - \bar{r}(I; \bar{I})$ rises with \bar{I} . Since $\underline{x} = \bar{r}(\bar{I}; \bar{I})$, we see that \underline{x} also falls as \bar{I} rises. Thus if $\bar{I}' > \bar{I}$, then $\underline{x}' < \underline{x}$.

Theorem 2. If the equilibrium verification and reporting policies are as described in Theorem 1, then for $\bar{I}' > \bar{I}$,

- (a) $I - \bar{r}(I; \bar{I}') > I - \bar{r}(I; \bar{I})$ for $I \in [\underline{I}, \bar{I}]$; and
- (b) $\bar{p}(x; \bar{I}') > \bar{p}(x; \bar{I})$ for $x \in (\underline{x}', \bar{x}')$; the equilibrium verification policies coincide outside this interval.

Figure 2 illustrates these results.

Proof. We can partition the interval $(-\infty, \infty)$ into five sets: $(-\infty, \underline{x}')$, $(\underline{x}', \underline{x}]$, (\underline{x}, \bar{x}) , (\bar{x}, \bar{x}') and (\bar{x}', ∞) . For $x \in (-\infty, \underline{x}')$, the optimal verification policies agree: $\bar{p}(x; \bar{I}') = \bar{p}(x; \bar{I}) = c'^{-1}(t(1 + \pi)(\bar{I} - x))$. For $x \in (\underline{x}', \underline{x}]$, $\bar{p}(x; \bar{I}') = p_0(x; \bar{I}') > c'^{-1}(t(1 + \pi)(\bar{I} - x)) = \bar{p}(x; \bar{I})$. The inequality follows from Lemma 2. For $x \in (\underline{x}, \bar{x})$, $\bar{p}(x; \bar{I}') = p_0(x; \bar{I}') > p_0(x; \bar{I}) = \bar{p}(x; \bar{I})$. For $x \in (\bar{x}, \bar{x}')$, $\bar{p}(x; \bar{I}') = p_0(x; \bar{I}') > 0 = \bar{p}(x; \bar{I})$. Finally, for $x \in (\bar{x}', \infty)$, the policies again agree, with $\bar{p}(x; \bar{I}') = \bar{p}(x; \bar{I}) = 0$.

Q.E.D.

That is, if two individuals have the same true income I , then that individual with the larger value of \bar{I} will fail to report a greater amount of income than the one with the smaller value of \bar{I} , and will accordingly find a greater amount of effort devoted to his investigation. Alternatively, the individual who resides in Beverly Hills, but reports the same income as a resident of Death Valley, will (be inferred to) have concealed more income, and is more likely to be investigated, than his counterpart who resides in

Death Valley.

VI. Costs of Verification

One important policy question in the area of tax compliance is: what is the impact of an increased capacity to verify income upon equilibrium verification and reporting policies? One could imagine such an increased capacity coming about through technical progress which uniformly lowers the cost of income verification, or through simplification of the tax form itself.

To model this question formally, let $k(p)$ represent another cost function which satisfies (A1)-(A3), with the property that $k'(p) < c'(p)$ for all $p \in [0,1]$. Since $k(0) = c(0) = 0$, it follows that $k(p) < c(p)$ for all $p \in (0,1]$.

Let $\bar{p}(x;k)$ and $\bar{p}(x;c)$ denote equilibrium verification policies under the cost functions $k(\cdot)$ and $c(\cdot)$, respectively. Let $\bar{x}_k = \bar{I} - k'(0)/(1 + \pi)$ and $\bar{x}_c = \bar{I} - c'(0)/(1 + \pi)$; clearly, $\bar{x}_k > \bar{x}_c$. Let $p_0(x;k)$ and $p_0(x;c)$ denote the solutions to equations (7) and (7') below, where (7) is restated here for easy reference.

$$-p'(x)c'(p(x)) + \pi p(x) - t(1 - p(x)) = 0 \quad (7)$$

$$-p'(x)k'(p(x)) + \pi p(x) - t(1 - p(x)) = 0. \quad (7')$$

Lemma 3. Suppose that $k(\cdot)$ and $c(\cdot)$ are two cost functions satisfying (A1)-(A3) with $k'(p) < c'(p)$ for all $p \in [0,1]$, and that $p_0(x;c)$ and $p_0(x;k)$ are solutions of equations (7) and (7'), respectively, which also satisfy condition (B). Then $p_0(x;k) > p_0(x;c)$ for all $x \leq \bar{x}_c$.

Proof. Since $p_0'(x;k) < 0$, $p_0(\bar{x}_k;k) = 0$ and $\bar{x}_c < \bar{x}_k$, it follows that $p_0(\bar{x}_c;k) > 0 = p_0(\bar{x}_c;c)$. In order for $p_0(x;k) \leq p_0(x;c)$ at some $x < \bar{x}_c$, it

must be that $p_0(x;c)$ crosses $p_0(x;k)$ from below at some point x_0 . At this point, it must be that $p_0'(x;c) \leq p_0'(x;k)$ (see Figure 3). From equations (7) and (7'), we see that at a crossing point (x_0, p_0) , it must be that $-p_0'(x_0;c)c'(p_0) = -p_0'(x_0;k)k'(p_0)$. Since $c'(p) > k'(p)$ for all p , it follows that $-p_0'(x;c) < -p_0'(x;k)$; that is, $p_0'(x;c) > p_0'(x;k)$. But this is a contradiction. Thus $p_0(x;k) > p_0(x;c)$ for all $x \leq \bar{x}_c$.

Q.E.D.

Unfortunately, we cannot determine unambiguously the impact of lower verification costs upon the equilibrium reporting policy $\bar{r}(I)$. To see why, note that

$$r_0^{-1}(x;c) = x + c'(p_0(x;c))/t(1 + \pi)$$

while

$$r_0^{-1}(x;k) = x + k'(p_0(x;k))/t(1 + \pi).$$

Switching from the cost function $c(\cdot)$ to the lower cost function $k(\cdot)$ has two effects; first, it lowers r_0^{-1} for the same value of p ; however, it simultaneously raises the equilibrium value of p , thus raising r_0^{-1} . The net result of these two conflicting effects is generally ambiguous. As a consequence, we cannot order the lower limits \bar{x}_k and \bar{x}_c , which are defined implicitly by the equations $\bar{I} = r_0^{-1}(\bar{x}_k;k)$ and $\bar{I} = r_0^{-1}(\bar{x}_c;c)$, respectively. Despite this ambiguity, it is possible to make the following comparison between $\bar{p}(x;k)$ and $\bar{p}(x;c)$.

Theorem 3. Suppose that $k(\cdot)$ and $c(\cdot)$ are two cost functions satisfying (A1)-(A3), with $k'(p) < c'(p)$ for all $p \in [0,1]$, and suppose that $p_0(x;c)$ and

$p_0(x;k)$ are solutions of (7) and (7'), respectively, which also satisfy condition (B). Then $\bar{p}(x;k) > \bar{p}(x;c)$ for all x . That is, if there is a uniform decrease in the marginal cost of verification, then greater effort will be devoted to verification.

Proof. For $x \in [\bar{x}_k, \infty)$, the two verification policies coincide, with $\bar{p}(x;k) = \bar{p}(x;c) = 0$. For $x \in [\bar{x}_c, \bar{x}_k)$, $\bar{p}(x;k) > 0 = \bar{p}(x;c)$. For $x \in (\max\{\bar{x}_k, \bar{x}_c\}, \bar{x}_c)$, $\bar{p}(x;k) = p_0(x;k) > p_0(x;c) = \bar{p}(x;c)$ by Lemma 3. For $x \in (-\infty, \min\{\bar{x}_k, \bar{x}_c\})$, $\bar{p}(x;k) = k'^{-1}(t(1 + \pi)(\underline{I} - x)) > c'^{-1}(t(1 + \pi)(\underline{I} - x)) = \bar{p}(x;c)$. Finally, we need to consider values of $x \in (\min\{\bar{x}_k, \bar{x}_c\}, \max\{\bar{x}_k, \bar{x}_c\})$. If $\bar{x}_k \geq \bar{x}_c$, then for $x \in (\bar{x}_c, \bar{x}_k]$, $\bar{p}(x;k) = k'^{-1}(t(1 + \pi)(\underline{I} - x)) \geq p_0(x;k) > p_0(x;c) = \bar{p}(x;c)$. The first inequality follows from Lemma 2, and the second inequality follows from Lemma 3. If $\bar{x}_k < \bar{x}_c$, then for $x \in (\bar{x}_k, \bar{x}_c]$ we have $\bar{p}(x;k) = p_0(x;k) > k'^{-1}(t(1 + \pi)(\underline{I} - x)) > c'^{-1}(t(1 + \pi)(\underline{I} - x)) = \bar{p}(x;c)$. The first inequality follows from Lemma 2, and the second from the assumption that $k'(\rho) < c'(\rho)$ for all $\rho \in [0,1]$. Thus $\bar{p}(x;k) > \bar{p}(x;c)$ for all x .

Q.E.D.

VII. Risk-Averse Taxpayers

Let us now reconsider the analysis of Section II under the assumption that taxpayers are risk-averse. Let $u(I)$ denote the utility the taxpayer receives from income I , and assume that $u'(I) > 0$ and $u''(I) < 0$ for all I . Then the taxpayer's expected utility when true income is I , he reports x , and is faced with the verification policy $p(\cdot)$, is

$$U(I, x; p) = p(x)u(I - tI - t\pi(I - x)) + (1 - p(x))u(I - tx).$$

The analogs of equations (2) and (3) are:

$$p'(x)[u(z_1) - u(z_2)] + p(x)u'(z_1)t\pi - t(1 - p(x))u'(z_2) = 0 \quad (10)$$

and

$$p''(x)[u(z_1) - u(z_2)] + 2p'(x)[u'(z_1)t\pi + u'(z_2)t] + p(x)u''(z_1)(t\pi)^2 + (1 - p(x))u''(z_2)t^2 \leq 0 \quad (11)$$

where $z_1 = I - tI - t\pi(I - x)$ and $z_2 = I - tx$.

If (τ_0, p_0, r_0) are part of a sequential equilibrium, and these policies satisfy equations (1'), (10) and (11) simultaneously, then the analysis of Lemma 1 can be repeated (under one additional assumption) to obtain the following result.

Lemma 4. If the inequality (11) is strict, and $1 - t(1 + \pi) \leq 0$, then $p_0'(x) < 0$ and $r_0^{-1}(x) \in (0,1)$.

Proof. Substituting $I = r_0^{-1}(x)$ and $\tau_0(x) = r_0^{-1}(x)$ into equations (1') and (10) yields

$$t(1 + \pi)(r_0^{-1}(x) - x) - c'(p_0) = 0 \quad (12)$$

$$p_0'(x)[u(\bar{z}_1) - u(\bar{z}_2)] + p_0(x)u'(\bar{z}_1)t\pi - t(1 - p_0(x))u'(\bar{z}_2) = 0, \quad (13)$$

where $\bar{z}_1 = r_0^{-1}(x)(1 - t) - t\pi(r_0^{-1}(x) - x)$ and $\bar{z}_2 = r_0^{-1}(x) - tx$.

Equations (12) and (13) must hold in a neighborhood of x when $p_0(x) \in (0,1)$. Differentiating with respect to x and collecting terms yields

$$t(1 + \pi)(r_0^{-1}(x) - 1) = c''(p_0)p_0'(x) \quad (14)$$

$$\begin{aligned}
& p_0''(x)[u(\bar{z}_1) - u(\bar{z}_2)] + 2p_0'(x)[u'(\bar{z}_1)t\pi + tu'(\bar{z}_2)] \\
& \quad + p_0(x)u''(\bar{z}_1)(t\pi)^2 + t^2(1 - p_0(x))u''(\bar{z}_2) \\
& = -r_0^{-1}'(x) \left\{ p_0'(x)[u'(\bar{z}_1)(1 - t(1 + \pi)) - u'(\bar{z}_2)] \right. \\
& \quad \left. + p_0(x)u''(\bar{z}_1)t\pi(1 - t(1 + \pi)) - t(1 - p_0(x))u''(\bar{z}_2) \right\}. \quad (15)
\end{aligned}$$

If $1 - t(1 + \pi) \leq 0$, then the expression $u'(\bar{z}_1)(1 - t(1 + \pi)) - u'(\bar{z}_2) < 0$, and the expression $p_0(x)u''(\bar{z}_1)t\pi(1 - t(1 + \pi)) - t(1 - p_0(x))u''(\bar{z}_2) > 0$. From (14), either (i) $r_0^{-1}'(x) - 1 > 0$ and $p_0'(x) > 0$ or (ii) $r_0^{-1}'(x) - 1 < 0$ and $p_0'(x) < 0$. Since the left-hand side of equation (15) is negative when inequality (11) is strict, either (iii) $r_0^{-1}'(x) < 0$ and $p_0'(x) > 0$, or (iv) $r_0^{-1}'(x) > 0$ and the bracketed term on the right-hand side of (15) > 0 . Conditions (i) and (iii) are contradictory, so conditions (ii) and (iv) must hold. That is, $p_0'(x) < 0$ and $r_0^{-1}'(x) \in (0, 1)$.

Q.E.D.

The parametric restriction $1 - t(1 + \pi) \leq 0$ is sufficient, but not necessary, for this proposition to be true (recall that risk neutrality on the part of the taxpayer was sufficient, without any additional parametric assumptions). One way to interpret this restriction is as follows: if not detected, a dollar of unreported income is worth \$1; if detected, the tax plus penalty paid on a dollar of unreported income is $\$(1 + \pi)$. Thus the assumption that $1 - t(1 + \pi) \leq 0$ means that the taxpayer's marginal (money) gain to under-reporting if he goes undetected is at least wiped out by his marginal loss if detected.

VIII. Conclusions

This paper presents a simple model of tax compliance as a game of incomplete information. The sequence of moves has the taxpayer first reporting his income, and the IRS subsequently acting optimally on the basis of this report. Thus the IRS is not permitted to make non-credible threats about its verification policy. We find that an equilibrium verification policy involves devoting greater resources to verification for those taxpayers reporting lower levels of income. The equilibrium reporting rule for taxpayers implies that taxpayers with greater income under-report less than those with lower income. We also find that although the statutory form of the tax schedule is linear, the effective tax schedule under incomplete information is regressive in the sense that the expected average tax rate declines as income rises.

It is important to emphasize that the aforementioned results are derived by considering only a single class of taxpayers; that is, under the assumption that one cannot distinguish among taxpayers ex ante on the basis of some other observable characteristic. When an observable characteristic which is related to ex ante opportunities for high income is available, we find that those classes of taxpayers with greater income opportunities (in terms of maximum possible income) fail to report a greater amount of their income and face harsher verification policies. That is, if two taxpayers have the same true income, then the one who ex ante enjoyed a better range of income opportunities will report less income; for any two taxpayers who report the same level of income, more effort will be devoted to income verification for the taxpayer who ex ante enjoyed a better range of income opportunities.

A uniform decrease in the marginal cost of verification leads to an increase in efforts devoted to verification, but the net effect of this change on equilibrium compliance is ambiguous.

These results have been derived under fairly stringent restrictions on the cost function $c(\cdot)$; in particular, we assumed that (A1), (A2) and (A3) held. The restriction (A2) is largely for simplicity; its elimination would require additional analysis of the Kuhn-Tucker conditions (1), which would indicate when and if the optimal reporting policy reaches the boundary $\bar{p}(x) = 1$. This would add another branch to the description of $\bar{p}(x)$, but is not likely to cause additional complications. Assumptions (A1) and (A3) are objectionable in that they jointly rule out the class of cost functions in which $c'(0) = 0$, and possibly other natural classes of cost functions. The former assumption was used to assert the uniqueness of $p_0(\cdot)$ through the given boundary condition, and again in the proof that the singular solution $\tilde{p}(x)$ does not generate a sequential equilibrium. The latter was used in the proof that $p_0(x)$ does generate a sequential equilibrium. A desirable extension of this work would include the elimination of these restrictions. Although this might result in multiple solutions to equation (7), for particular cases one can always directly verify or reject these solutions as sequential equilibria.

FOOTNOTES

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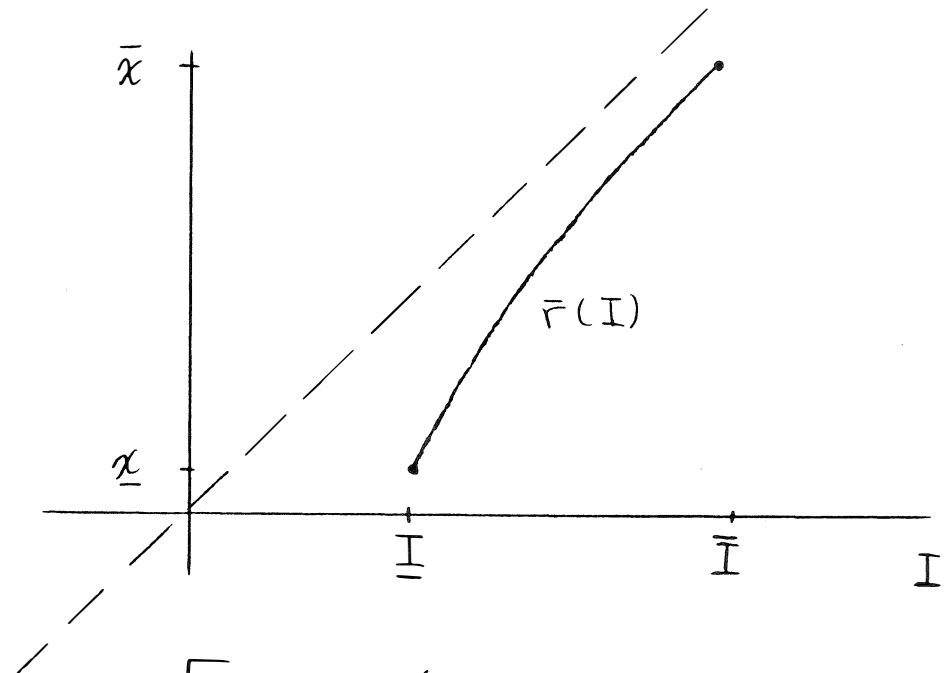
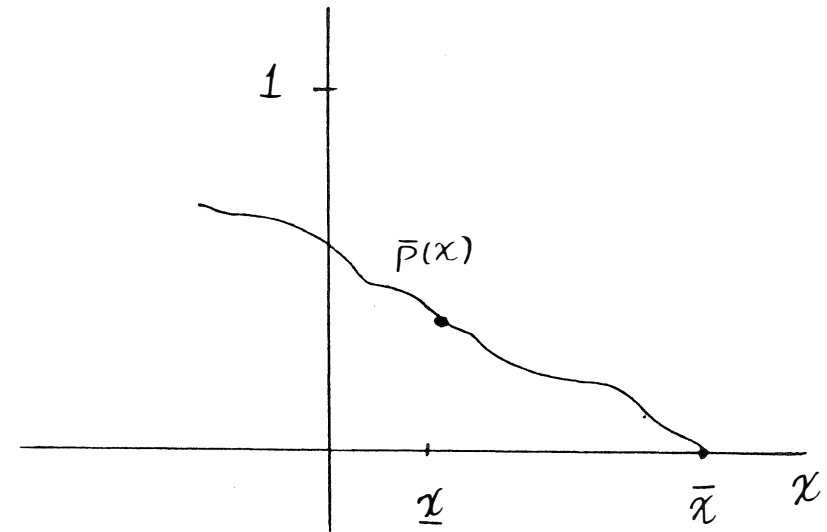


Figure 1

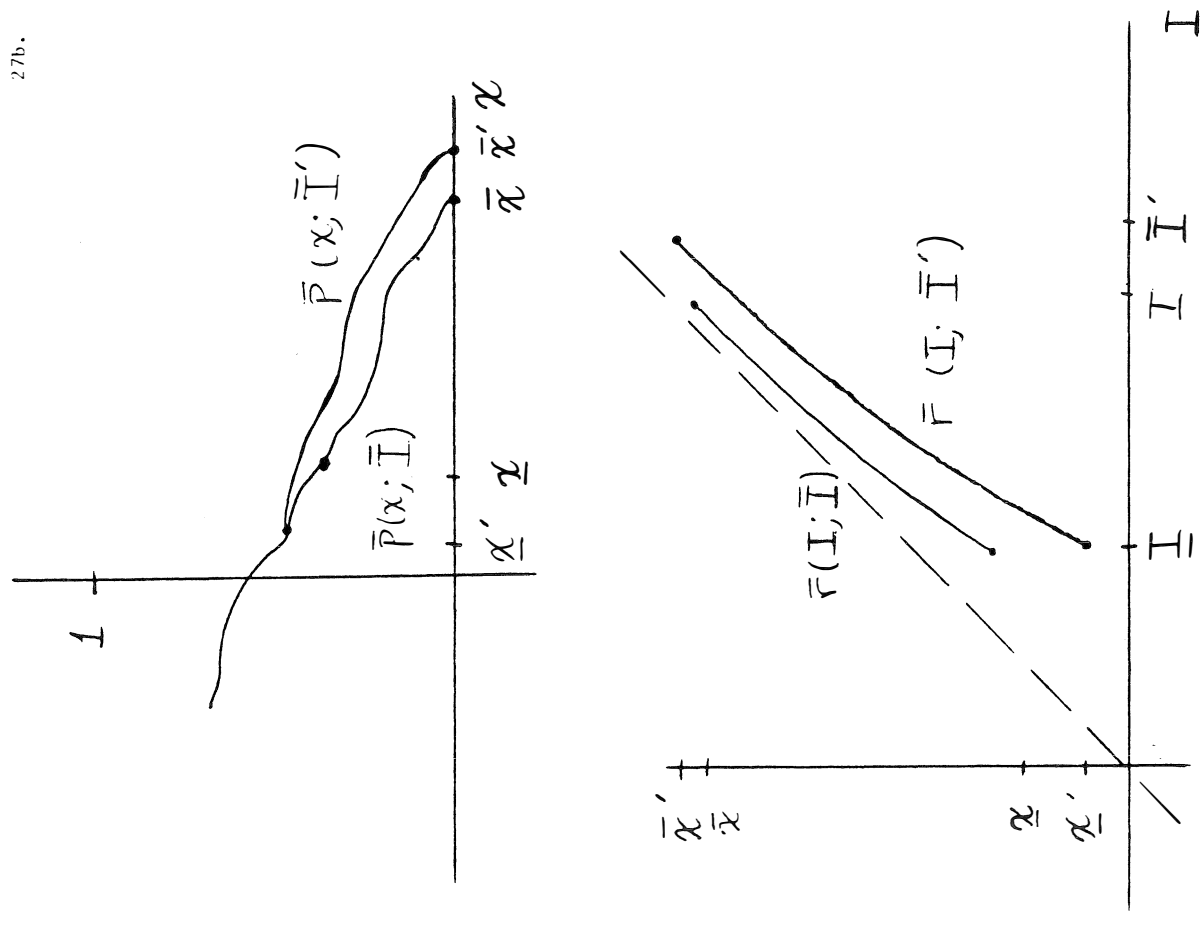


Figure 2

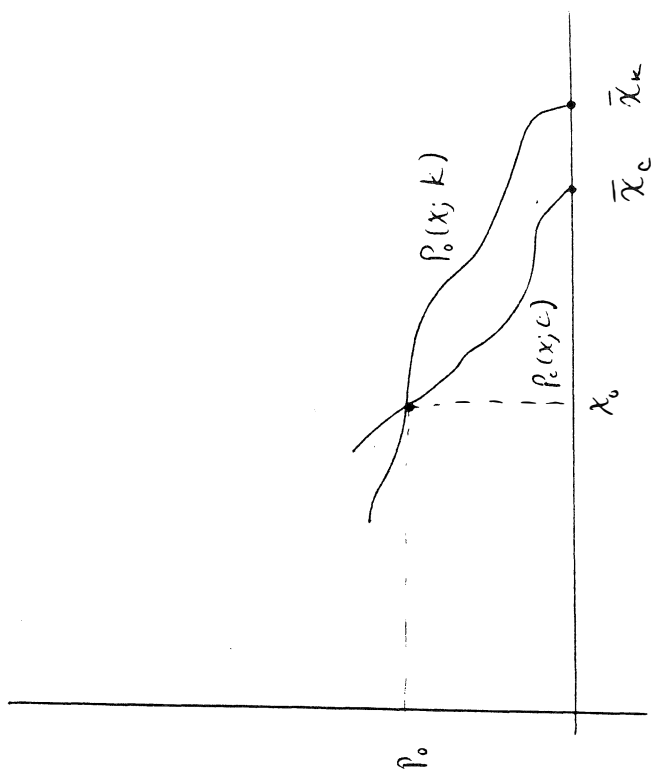


Figure 3

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APPENDIX

In this appendix, we verify Corollary 2 directly.

Proof. Recall that $\bar{r}(I) = r_0(I)$; thus $\bar{r}(\bar{I}) = \bar{x}$ by definition. Since r_0^{-1} is invertible on $[\underline{x}, \bar{x}]$, $\bar{r}(\bar{I}) = \bar{x}$ if and only if $r_0^{-1}(\bar{x}) = \bar{I}$.

$$r_0^{-1}(x) = x + \frac{1 + \pi - \exp[-(t\pi/c)(\bar{x} - x)]}{(t\pi/c)(1 + \pi)} = f(x).$$

Evaluating at \bar{x} implies that $r_0^{-1}(\bar{x}) = \bar{I}$. The inversion of r_0^{-1} is justified since $r_0^{-1}'(x) = f'(x) > 0$ for all $x < \bar{x}$. Thus $\bar{\tau}(x)$ satisfies the consistency requirement.

Next we show that given $\bar{\tau}(\cdot)$, $\bar{p}(x)$ maximizes $R(x, p; \bar{\tau})$.

For $x \in [\bar{x}, \infty)$, $\bar{\tau}(x) = \bar{I}$, and

$$R(x, p; \bar{\tau}) = p[t\bar{I} + t\pi(\bar{I} - x)] + (1 - p)tx - c \ln(1 - p).$$

Differentiating with respect to p and evaluating at $p = 0$ implies that

$$R_p(x, 0; \bar{\tau}) = t(1 + \pi)(\bar{I} - x) - c \leq 0$$

for all $x \geq \bar{x}$, so $\bar{p}(x) = 0$ is optimal for $x \geq \bar{x}$.

For $x \in [\underline{x}, \bar{x}]$, $\bar{\tau}(x) = r_0^{-1}(x)$ and

$$R(x, p; \bar{\tau}) = p[tr_0^{-1}(x) + \pi t(r_0^{-1}(x) - x)] + (1 - p)tx - c \ln(1 - p).$$

Differentiation and evaluation at $p = 0$ implies that

$$\begin{aligned} R_p(x, 0; \bar{\tau}) &= t(1 + \pi)(r_0^{-1}(x) - x) - c \\ &= (1/\pi)[1 + \pi - \exp[-t\pi(\bar{x} - x)]] - c > 0 \end{aligned}$$

for all $x \in [\underline{x}, \bar{x}]$. Thus $\bar{p}(x) \in (0,1)$ for $x \in [\underline{x}, \bar{x}]$. Solving $R_\rho(x, \bar{p}(x); \tau) = 0$ for $\bar{p}(x)$ yields

$$\bar{p}(x) = 1 - \pi / [1 + \pi - \exp\{-(t\pi/c)(\bar{x} - x)\}].$$

For $x \leq \underline{x}$, $\bar{\tau}(x) = \underline{I}$. Since $R_\rho(\bar{r}(\underline{I}), 0; \tau) > 0$ from the previous argument, and since

$$R_\rho(x, 0; \tau) = t(1 + \pi)(\underline{I} - x) - c$$

is decreasing in x , it follows that $R_\rho(x, 0; \tau) > 0$ for all $x < \underline{x} = \bar{r}(\underline{I})$. Thus $\bar{p}(x)$ is interior. Solving $R_\rho(x, \bar{p}(x); \tau) = 0$ yields

$$\bar{p}(x) = 1 - c/t(1 + \pi)(\underline{I} - x), \quad \text{for } x \leq \underline{x}.$$

Note that $\bar{p}(x)$ is a continuous function of x .

Finally, it must be shown that $\bar{r}(\bar{I})$ is best against $\bar{p}(x)$. $N_x(I, x; \bar{p})$ is a continuous function of x because $\bar{p}(x)$ is continuous. It is clearly suboptimal for any type I to report $x > \bar{x}$; any such report is dominated by a report of \bar{x} . Similarly, any report of $x < \underline{x}$ is dominated by a report of \underline{x} . To see this, note that for $x < \underline{x}$, $N(I, x; \bar{p})$ is differentiable with

$$N_x(I, x; \bar{p}) = [1/(\underline{I} - x)][c(I - x)/(\underline{I} - x) + t\pi(\underline{I} - x) - c].$$

$N_x(\underline{I}, x; \bar{p}) = t\pi > 0$ for all $x < \underline{x}$. Since N_x is increasing in I , $N_x(I, x; \bar{p}) > 0$ for all $x < \underline{x}$ and all $I \in [\underline{I}, \bar{I}]$. Thus a report of \underline{x} is preferred to any report $x < \underline{x}$. Finally, for $x \in (\underline{x}, \bar{x})$,

$$N_x(I, x; \bar{p}) = \left\{ \frac{t\pi \exp\{-(t\pi/c)(\bar{x} - x)\}}{1 + \pi - \exp\{-(t\pi/c)(\bar{x} - x)\}} \right\} \times \left\{ \frac{(t\pi/c)(1 + \pi)(I - x)}{1 + \pi - \exp\{-(t\pi/c)(\bar{x} - x)\}} - 1 \right\}.$$

$N_x(I, \bar{x}; \bar{p}) < 0$, except for $I = \bar{I}$, in which case $N_x(\bar{I}, \bar{x}; \bar{p}) = 0$ by the definition of \bar{x} . Similarly, $N_x(I, \underline{x}; \bar{p}) > 0$ except when $I = \underline{I}$, in which case $N_x(\underline{I}, \underline{x}; \bar{p}) = 0$ by the definition of \underline{x} . Thus $\bar{r}(I) \in (\underline{x}, \bar{x})$ for each $I \in (\underline{I}, \bar{I})$. Solving $N_x(I, \bar{r}(I); \bar{p}) = 0$ for $\bar{r}(I)$ implies that $x = \bar{r}(I)$ is the unique value of $x \in (-\infty, \bar{x}]$ such that

$$I - \bar{r}(I) = [1 + \pi - \exp\{-(t\pi/c)(\bar{x} - \bar{r}(I))\}]/(t\pi/c)(1 + \pi).$$

Q.E.D.